MAXWELL INTEGRAL QUATERNION QUANTIZED SPACETIME EVOLUTION

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Abstract. Picking up Peter Jack's arxiv:math-ph/0307038 left-right-balanced quaternionic reformulation of Maxwellian physics as a

$$\phi \in \{\phi \in (\mathbb{R}^4 \to \mathbb{R}^4) \mid \{\{\phi, \nabla\}, \nabla\}\} + [[\phi, \nabla], \nabla] = [0, 0, 0, 0]\}$$

Roman Czyborra presents how this leads to a countable and computable deterministic evolution if the spacetime location fabric is latticed as $\frac{1}{2}\mathbb{Z}^4$.

Recall that the quaternions $\vec{p} \in \mathbb{Q}^4$ and $\vec{q} \in \mathbb{Q}^4$ are left-right-multiplied as

$$\vec{p} \times \vec{q} = \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} \times \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} := \begin{bmatrix} +p_0 & -p_1 & -p_2 & -p_3 \\ -p_1 & +p_0 & -p_3 & +p_2 \\ -p_2 & +p_3 & +p_0 & -p_1 \\ -p_3 & -p_2 & +p_1 & +p_0 \end{bmatrix} \times \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

which can be split up into a symmetrically commuting product component

$$\{\vec{p}, \vec{q}\} := \frac{[\vec{p} \times \vec{q}] + [\vec{q} \times \vec{p}]}{2} = \begin{bmatrix} +p_0 & -p_1 & -p_2 & -p_3 \\ -p_1 & +p_0 & 0 & 0 \\ -p_2 & 0 & +p_0 & 0 \\ -p_3 & 0 & 0 & +p_0 \end{bmatrix} \times \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

and an order-sensitive antisymmetric anticommuting product component

$$[\vec{p}, \vec{q}] := \frac{[\vec{p} \times \vec{q}] - [\vec{q} \times \vec{p}]}{2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -p_3 & +p_2 \\ 0 & +p_3 & 0 & -p_1 \\ 0 & -p_2 & +p_1 & 0 \end{bmatrix} \times \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

whose Gâteaux nabla operator $\vec{\nabla} \in [[\mathbb{R}^4 \to \mathbb{R}^4] \to [\mathbb{R}^4 \to \mathbb{R}^4]]$ introduced as

$$\left[\vec{\nabla} \times \vec{q}\right][\vec{v}] := \lim_{d \to \pm 0} \sum_{i \in \{0,1,2,3\}} [d\vec{e}_i]^{-1} \times [\vec{q}\,[\vec{v} + d\vec{e}_i] - \vec{q}\,[\vec{v}]]$$

$$\left[\vec{q} \times \vec{\nabla}\right][\vec{v}] := \lim_{d \to \pm 0} \sum_{i \in \{0,1,2,3\}} [\vec{q}\,[\vec{v} + d\vec{e}_i] - \vec{q}\,[\vec{v}]] \times [d\vec{e}_i]^{-1}$$

shall now under the possibly oversimplifying assumption that an underlying field $\frac{1}{2}\mathbb{Z}^4$ exposes no smaller positive distance than $d \to \min \frac{1}{2}\mathbb{Z}^+ = \frac{1}{2}$ be redefined as

$$[\vec{\nabla} \times \vec{q}][\vec{v}] := \sum_{i \in \{0,1,2,3\}} [\vec{e}_i]^{-1} \times \left[\vec{q}\left[\vec{v} + \frac{\vec{e}_i}{2}\right] - \vec{q}\left[\vec{v} - \frac{\vec{e}_i}{2}\right]\right]$$

$$[\vec{q} \times \vec{\nabla}][\vec{v}] := \sum_{i \in \{0,1,2,3\}} \left[\vec{q}\left[\vec{v} + \frac{\vec{e}_i}{2}\right] - \vec{q}\left[\vec{v} - \frac{\vec{e}_i}{2}\right]\right] \times [\vec{e}_i]^{-1}$$

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such that, using the deltas

$$\vec{q}\vec{\Delta}_0 \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} := \vec{q} \begin{bmatrix} w+\frac{1}{2} \\ x+0 \\ y+0 \\ z+0 \end{bmatrix} - \vec{q} \begin{bmatrix} w-\frac{1}{2} \\ x-0 \\ y-0 \\ z-0 \end{bmatrix} = \begin{bmatrix} q_0[w+\frac{1}{2},x,y,z] - q_0[w-\frac{1}{2},x,y,z] \\ q_1[w+\frac{1}{2},x,y,z] - q_1[w-\frac{1}{2},x,y,z] \\ q_2[w+\frac{1}{2},x,y,z] - q_2[w-\frac{1}{2},x,y,z] \\ q_3[w+\frac{1}{2},x,y,z] - q_3[w-\frac{1}{2},x,y,z] \end{bmatrix}$$

and

$$\vec{q}\vec{\Delta}_1 \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} := \vec{q} \begin{bmatrix} w+0 \\ x+\frac{1}{2} \\ y+0 \\ z+0 \end{bmatrix} - \vec{q} \begin{bmatrix} w-0 \\ x-\frac{1}{2} \\ y-0 \\ z-0 \end{bmatrix} = \begin{bmatrix} q_0[w,x+\frac{1}{2},y,z] - q_0[w,x-\frac{1}{2},y,z] \\ q_1[w,x+\frac{1}{2},y,z] - q_1[w,x-\frac{1}{2},y,z] \\ q_2[w,x+\frac{1}{2},y,z] - q_2[w,x-\frac{1}{2},y,z] \\ q_3[w,x+\frac{1}{2},y,z] - q_3[w,x-\frac{1}{2},y,z] \end{bmatrix}$$

and

$$\vec{q}\vec{\Delta}_2 \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} := \vec{q} \begin{bmatrix} w+0 \\ x+0 \\ y+\frac{1}{2} \\ z+0 \end{bmatrix} - \vec{q} \begin{bmatrix} w-0 \\ x-0 \\ y-\frac{1}{2} \\ z-0 \end{bmatrix} = \begin{bmatrix} q_0[w,x,y+\frac{1}{2},z] - q_0[w,x,y-\frac{1}{2},z] \\ q_1[w,x,y+\frac{1}{2},z] - q_1[w,x,y-\frac{1}{2},z] \\ q_2[w,x,y+\frac{1}{2},z] - q_2[w,x,y-\frac{1}{2},z] \\ q_3[w,x,y+\frac{1}{2},z] - q_3[w,x,y-\frac{1}{2},z] \end{bmatrix}$$

and

$$\vec{q}\vec{\Delta}_3 \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} := \vec{q} \begin{bmatrix} w+0 \\ x+0 \\ y+0 \\ z+\frac{1}{2} \end{bmatrix} - \vec{q} \begin{bmatrix} w-0 \\ x-0 \\ y-0 \\ z-\frac{1}{2} \end{bmatrix} = \begin{bmatrix} q_0[w,x,y,z+\frac{1}{2}] - q_0[w,x,y,z-\frac{1}{2}] \\ q_1[w,x,y,z+\frac{1}{2}] - q_1[w,x,y,z-\frac{1}{2}] \\ q_2[w,x,y,z+\frac{1}{2}] - q_2[w,x,y,z-\frac{1}{2}] \\ q_3[w,x,y,z+\frac{1}{2}] - q_3[w,x,y,z-\frac{1}{2}] \end{bmatrix}$$

in their directional contributions

$$[1,0,0,0]^{-1} \times [d_0,d_1,d_2,d_3] = [1,0,0,0] \times [d_0,d_1,d_2,d_3] = [d_0,d_1,d_2,d_3]$$

$$[d_0,d_1,d_2,d_3] \times [1,0,0,0]^{-1} = [d_0,d_1,d_2,d_3] \times [1,0,0,0] = [d_0,d_1,d_2,d_3]$$

$$\{[1,0,0,0],[d_0,d_1,d_2,d_3]\} = [d_0,d_1,d_2,d_3]$$

$$[[1,0,0,0],[d_0,d_1,d_2,d_3]] = [0,0,0,0]$$

and

$$[0,1,0,0]^{-1} \times [d_0,d_1,d_2,d_3] = [0,-1,0,0] \times [d_0,d_1,d_2,d_3] = [d_1,-d_0,d_3,-d_2]$$

$$[d_0,d_1,d_2,d_3] \times [0,1,0,0]^{-1} = [d_0,d_1,d_2,d_3] \times [0,-1,0,0] = [d_1,-d_0,-d_3,d_2]$$

$$\{[0,-1,0,0],[d_0,d_1,d_2,d_3]\} = [d_1,-d_0,0,0]$$

$$[[0,-1,0,0],[d_0,d_1,d_2,d_3]] = [0,0,d_3,-d_2]$$

and

$$[0,0,1,0]^{-1} \times [d_0,d_1,d_2,d_3] = [0,0,-1,0] \times [d_0,d_1,d_2,d_3] = [d_2,-d_3,-d_0,d_1]$$

$$[d_0,d_1,d_2,d_3] \times [0,0,1,0]^{-1} = [d_0,d_1,d_2,d_3] \times [0,0,-1,0] = [d_2,d_3,-d_0,-d_1]$$

$$\{[0,0,-1,0],[d_0,d_1,d_2,d_3]\} = [d_2,0,-d_0,0]$$

$$[[0,0,-1,0],[d_0,d_1,d_2,d_3]] = [0,-d_3,0,d_1]$$

and

$$[0,0,0,1]^{-1} \times [d_0,d_1,d_2,d_3] = [0,0,0,-1] \times [d_0,d_1,d_2,d_3] = [d_3,d_2,-d_1,-d_0]$$

$$[d_0,d_1,d_2,d_3] \times [0,0,0,1]^{-1} = [d_0,d_1,d_2,d_3] \times [0,0,0,-1] = [d_3,-d_2,d_1,-d_0]$$

$$\{[0,0,0,-1],[d_0,d_1,d_2,d_3]\} = [d_3,0,0,-d_0]$$

$$[[0,0,0,-1],[d_0,d_1,d_2,d_3]] = [0,d_2,-d_1,0]$$

we get

$$\{\{\vec{p},\vec{\nabla}\},\vec{\nabla}\}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sum\left\{\begin{matrix} p_0[w+1,x,y,z]+4p_0[w,x,y,z]+p_0[w-1,x,y,z], \\ -p_0[w,x,y,z-1]-p_0[w,x,y-1,z]-p_0[w,x-1,y,z], \\ -p_0[w,x,y,z+1]-p_0[w,x,y+1,z]-p_0[w,x+1,y,z], \\ 2p_1[w+\frac{1}{2},x+\frac{1}{2},y,z]-2p_1[w+\frac{1}{2},x-\frac{1}{2},y,z], \\ 2p_1[w-\frac{1}{2},x-\frac{1}{2},y,z]-2p_1[w-\frac{1}{2},x+\frac{1}{2},y,z], \\ 2p_2[w+\frac{1}{2},x,y+\frac{1}{2},z]-2p_2[w+\frac{1}{2},x,y-\frac{1}{2},z], \\ 2p_2[w-\frac{1}{2},x,y-\frac{1}{2},z]-2p_2[w-\frac{1}{2},x,y+\frac{1}{2},z], \\ 2p_3[w+\frac{1}{2},x,y,z+\frac{1}{2}]-2p_3[w+\frac{1}{2},x,y,z-\frac{1}{2}], \\ 2p_3[w-\frac{1}{2},x,y,z-\frac{1}{2}]-2p_3[w-\frac{1}{2},x,y,z+\frac{1}{2}], \end{matrix}\right\} \\ \sum\left\{\begin{matrix} 2p_0[w+\frac{1}{2},x-\frac{1}{2},y,z]-2p_0[w+\frac{1}{2},x+\frac{1}{2},y,z], \\ 2p_0[w-\frac{1}{2},x+\frac{1}{2},y,z]-2p_0[w-\frac{1}{2},x-\frac{1}{2},y,z], \\ p_1[w-1,x,y,z]-p_1[w,x+1,y,z], \\ p_1[w+1,x,y,z]-p_1[w,x-1,y,z], \\ p_2[w,x+\frac{1}{2},y-\frac{1}{2},z]-p_2[w,x+\frac{1}{2},y+\frac{1}{2},z], \\ p_2[w,x-\frac{1}{2},y+\frac{1}{2},z]-p_2[w,x-\frac{1}{2},y-\frac{1}{2},z], \\ p_3[w,x+\frac{1}{2},y,z-\frac{1}{2}]-p_3[w,x+\frac{1}{2},y,z+\frac{1}{2}], \\ p_3[w,x-\frac{1}{2},y,z+\frac{1}{2}]-p_3[w,x-\frac{1}{2},y,z-\frac{1}{2}], \end{matrix}\right\} \\ \sum\left\{\begin{matrix} 2p_0[w+\frac{1}{2},x,y-\frac{1}{2},z]-2p_0[w+\frac{1}{2},x,y+\frac{1}{2},z], \\ 2p_0[w-\frac{1}{2},x,y+\frac{1}{2},z]-2p_0[w-\frac{1}{2},x,y-\frac{1}{2},z], \\ p_1[w,x+\frac{1}{2},y-\frac{1}{2},z]-p_1[w,x+\frac{1}{2},y+\frac{1}{2},z], \\ p_1[w,x-\frac{1}{2},y+\frac{1}{2},z]-p_1[w,x-\frac{1}{2},y-\frac{1}{2},z], \\ p_2[w+1,x,y,z]-p_2[w,x,y+1,z], \\ p_2[w-1,x,y,z]-p_2[w,x,y-1,z], \\ p_3[w,x,y+\frac{1}{2},z-\frac{1}{2}]-p_3[w,x,y+\frac{1}{2},z+\frac{1}{2}], \\ p_3[w,x,y-\frac{1}{2},z+\frac{1}{2}]-p_3[w,x,y-\frac{1}{2},z-\frac{1}{2}], \end{matrix}\right\} \\ \sum\left\{\begin{matrix} 2p_0[w+\frac{1}{2},x,y,z-\frac{1}{2}]-2p_0[w+\frac{1}{2},x,y,z+\frac{1}{2}], \\ 2p_0[w-\frac{1}{2},x,y,z+\frac{1}{2}]-2p_0[w-\frac{1}{2},x,y,z-\frac{1}{2}], \\ p_1[w,x+\frac{1}{2},y,z-\frac{1}{2}]-p_1[w,x+\frac{1}{2},y,z+\frac{1}{2}], \\ p_1[w,x-\frac{1}{2},y,z+\frac{1}{2}]-p_1[w,x-\frac{1}{2},y,z-\frac{1}{2}], \\ p_2[w,x,y+\frac{1}{2},z-\frac{1}{2}]-p_2[w,x,y+\frac{1}{2},z+\frac{1}{2}], \\ p_2[w,x,y-\frac{1}{2},z+\frac{1}{2}]-p_2[w,x,y-\frac{1}{2},z-\frac{1}{2}], \\ p_3[w+1,x,y,z]-p_3[w,x,y,z+1], \\ p_3[w-1,x,y,z]-p_3[w,x,y,z-1] \end{matrix}\right\} \end{bmatrix}$$

as a shifted self-application of

$$\{\vec{q},\vec{\nabla}\}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sum\left\{\begin{matrix} q_0[w+\frac{1}{2},x,y,z]-q_0[w-\frac{1}{2},x,y,z], \\ q_1[w,x+\frac{1}{2},y,z]-q_1[w,x-\frac{1}{2},y,z], \\ q_2[w,x,y+\frac{1}{2},z]-q_2[w,x,y-\frac{1}{2},z], \\ q_3[w,x,y,z+\frac{1}{2}]-q_3[w,x,y,z-\frac{1}{2}] \end{matrix}\right\} \\ \sum\left\{\begin{matrix} q_0[w,x-\frac{1}{2},y,z]-q_0[w,x+\frac{1}{2},y,z], \\ q_1[w+\frac{1}{2},x,y,z]-q_1[w-\frac{1}{2},x,y,z] \end{matrix}\right\} \\ \sum\left\{\begin{matrix} q_0[w,x,y-\frac{1}{2},z]-q_0[w,x,y+\frac{1}{2},z], \\ q_2[w+\frac{1}{2},x,y,z]-q_2[w-\frac{1}{2},x,y,z] \end{matrix}\right\} \\ \sum\left\{\begin{matrix} q_0[w,x,y,z-\frac{1}{2}]-q_0[w,x,y,z+\frac{1}{2}], \\ q_3[w+\frac{1}{2},x,y,z]-q_3[w-\frac{1}{2},x,y,z] \end{matrix}\right\} \end{bmatrix}$$

and likewise the antisymmetric

$$[[\vec{p},\vec{\nabla}],\vec{\nabla}]\begin{bmatrix} w\\ x\\ y\\ z\end{bmatrix} = \begin{bmatrix} 0 \\ \sum\left\{\begin{matrix} 4p_1[w,x,y,z], \\ -p_1[w,x,y+1,z]-p_1[w,x,y,z+1], \\ -p_1[w,x,y-1,z]-p_1[w,x,y,z-1], \\ p_2[w,x+\frac{1}{2},y+\frac{1}{2},z]-p_2[w,x-\frac{1}{2},y+\frac{1}{2},z], \\ p_2[w,x-\frac{1}{2},y-\frac{1}{2},z]-p_2[w,x+\frac{1}{2},y-\frac{1}{2},z], \\ p_3[w,x+\frac{1}{2},y,z+\frac{1}{2}]-p_3[w,x+\frac{1}{2},y,z-\frac{1}{2}], \\ p_3[w,x-\frac{1}{2},y,z-\frac{1}{2}]-p_3[w,x-\frac{1}{2},y,z+\frac{1}{2}] \end{matrix}\right\} \\ \sum\left\{\begin{matrix} p_1[w,x+\frac{1}{2},y+\frac{1}{2},z]-p_1[w,x+\frac{1}{2},y-\frac{1}{2},z], \\ p_1[w,x-\frac{1}{2},y-\frac{1}{2},z]-p_1[w,x-\frac{1}{2},y+\frac{1}{2},z], \\ 4p_2[w,x,y,z], \\ -p_2[w,x+1,y,z]-p_2[w,x,y,z+1],-p_2[w,x-1,y,z]-p_2[w,x,y,z-1], \\ p_3[w,x,y+\frac{1}{2},z-\frac{1}{2}]-p_3[w,x,y+\frac{1}{2},z-\frac{1}{2}], \\ p_3[w,x,y-\frac{1}{2},z-\frac{1}{2}]-p_3[w,x,y-\frac{1}{2},z-\frac{1}{2}] \end{matrix}\right\} \\ \sum\left\{\begin{matrix} p_1[w,x+\frac{1}{2},y,z+\frac{1}{2}]-p_1[w,x+\frac{1}{2},y,z-\frac{1}{2}], \\ p_1[w,x-\frac{1}{2},y,z-\frac{1}{2}]-p_1[w,x-\frac{1}{2},y,z+\frac{1}{2}], \\ p_2[w,x,y+\frac{1}{2},z+\frac{1}{2}]-p_2[w,x,y+\frac{1}{2},z-\frac{1}{2}], \\ p_2[w,x,y-\frac{1}{2},z-\frac{1}{2}]-p_2[w,x,y-\frac{1}{2},z+\frac{1}{2}], \\ 4p_3[w,x,y,z], \\ -p_3[w,x+1,y,z]-p_3[w,x,y+1,z], \\ -p_3[w,x,y-1,z]-p_3[w,x-1,y,z], \end{matrix}\right\} \end{bmatrix}$$

as a shifted self-application of

$$[\vec{q},\vec{\nabla}]\begin{bmatrix} w\\ x\\ y\\ z\end{bmatrix} = \begin{bmatrix} 0 \\ \sum\left\{\begin{matrix} q_2[w,x,y,z+\frac{1}{2}]-q_2[w,x,y,z-\frac{1}{2}], \\ q_3[w,x,y-\frac{1}{2},z]-q_3[w,x,y+\frac{1}{2},z] \end{matrix}\right\} \\ \sum\left\{\begin{matrix} q_1[w,x,y,z-\frac{1}{2}]-q_1[w,x,y,z+\frac{1}{2}], \\ q_3[w,x+\frac{1}{2},y,z]-q_3[w,x-\frac{1}{2},y,z] \end{matrix}\right\} \\ \sum\left\{\begin{matrix} q_1[w,x,y+\frac{1}{2},z]-q_1[w,x,y-\frac{1}{2},z], \\ q_2[w,x-\frac{1}{2},y,z]-q_2[w,x+\frac{1}{2},y,z] \end{matrix}\right\} \end{bmatrix}$$

yielding, when demanding the Maxwell balance of matter-like local thermoelectric deceleration and local spatial curvature to even out

$$\{\{\vec{\phi},\vec{\nabla}\},\vec{\nabla}\}+[[\vec{\phi},\vec{\nabla}],\vec{\nabla}]=\vec{0}$$

a deterministic time evolution

$$\vec{\phi}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sum\left\{\begin{matrix} \phi_0[w-1,x+1,y,z]+\phi_0[w-1,x,y+1,z]+\phi_0[w-1,x,y,z+1], \\ \phi_0[w-1,x-1,y,z]+\phi_0[w-1,x,y-1,z]+\phi_0[w-1,x,y,z-1], \\ -4\phi_0[w-1,x,y,z]-\phi_0[w-2,x,y,z], \\ 2\phi_1[w-\frac{1}{2},x-\frac{1}{2},y,z]-2\phi_1[w-\frac{1}{2},x+\frac{1}{2},y,z], \\ 2\phi_1[w-\frac{3}{2},x+\frac{1}{2},y,z]-2\phi_1[w-\frac{3}{2},x-\frac{1}{2},y,z], \\ 2\phi_2[w-\frac{1}{2},x,y-\frac{1}{2},z]-2\phi_2[w-\frac{1}{2},x,y+\frac{1}{2},z], \\ 2\phi_2[w-\frac{3}{2},x,y+\frac{1}{2},z]-2\phi_2[w-\frac{3}{2},x,y-\frac{1}{2},z], \\ 2\phi_3[w-\frac{1}{2},x,y,z-\frac{1}{2}]-2\phi_3[w-\frac{1}{2},x,y,z+\frac{1}{2}], \\ 2\phi_3[w-\frac{3}{2},x,y,z+\frac{1}{2}]-2\phi_3[w-\frac{3}{2},x,y,z-\frac{1}{2}], \end{matrix}\right\} \\ \sum\left\{\begin{matrix} 2\phi_0[w-\frac{1}{2},x+\frac{1}{2},y,z]-2\phi_0[w-\frac{1}{2},x-\frac{1}{2},y,z], \\ 2\phi_0[w-\frac{3}{2},x-\frac{1}{2},y,z]-2\phi_0[w-\frac{3}{2},x+\frac{1}{2},y,z], \\ \phi_1[w-1,x+1,y,z]+\phi_1[w-1,x,y+1,z]+\phi_1[w-1,x,y,z+1], \\ \phi_1[w-1,x-1,y,z]+\phi_1[w-1,x,y-1,z]+\phi_1[w-1,x,y,z-1], \\ -4\phi_1[w-1,x,y,z]-\phi_1[w-2,x,y,z] \end{matrix}\right\} \\ \sum\left\{\begin{matrix} 2\phi_0[w-\frac{1}{2},x,y+\frac{1}{2},z]-2\phi_0[w-\frac{1}{2},x,y-\frac{1}{2},z], \\ 2\phi_0[w-\frac{3}{2},x,y-\frac{1}{2},z]-2\phi_0[w-\frac{3}{2},x,y+\frac{1}{2},z], \\ \phi_2[w-1,x+1,y,z]+\phi_2[w-1,x,y+1,z]+\phi_2[w-1,x,y,z+1], \\ \phi_2[w-1,x-1,y,z]+\phi_2[w-1,x,y-1,z]+\phi_2[w-1,x,y,z-1], \\ -4\phi_2[w-1,x,y,z]-\phi_2[w-2,x,y,z] \end{matrix}\right\} \\ \sum\left\{\begin{matrix} 2\phi_0[w-\frac{1}{2},x,y,z+\frac{1}{2}]-2\phi_0[w-\frac{1}{2},x,y,z-\frac{1}{2}], \\ 2\phi_0[w-\frac{3}{2},x,y,z-\frac{1}{2}]-2\phi_0[w-\frac{3}{2},x,y,z+\frac{1}{2}], \\ \phi_3[w-1,x+1,y,z]+\phi_3[w-1,x,y+1,z]+\phi_3[w-1,x,y,z+1], \\ \phi_3[w-1,x-1,y,z]+\phi_3[w-1,x,y-1,z]+\phi_3[w-1,x,y,z-1], \\ -4\phi_3[w-1,x,y,z]-\phi_3[w-2,x,y,z], \end{matrix}\right\} \end{bmatrix}$$

that can easily be simulated on a computer as in:

```
module Czyborra.Czyborspace20160901 (q) where
import Data.Function.Memoize (memoFix)
q=memoFix$ \q(v,w,x,y,z)->if(w>0)then case v of
 0->q(0,w-2,x+2,y,z)+q(0,w-2,x,y+2,z)+q(0,w-2,x,y,z+2)+
    q(0,w-2,x-2,y,z)+q(0,w-2,x,y-2,z)+q(0,w-2,x,y,z-2)+
    (0-4)*q(0,w-2,x,y,z)-q(0,w-4,x,y,z)+
    2*q(1,w-1,x-1,y,z)-2*q(1,w-1,x+1,y,z)+
    2*q(1,w-3,x+1,y,z)-2*q(1,w-3,x-1,y,z)+
    2*q(2,w-1,x,y-1,z)-2*q(2,w-1,x,y+1,z)+
    2*q(2,w-3,x,y+1,z)-2*q(2,w-3,x,y-1,z)+
    2*q(3,w-1,x,y,z-1)-2*q(3,w-1,x,y,z+1)+
    2*q(3,w-3,x,y,z+1)-2*q(3,w-3,x,y,z-1)
 1->2*q(0,w-1,x+1,y,z)-2*q(0,w-1,x-1,y,z)+
    2*q(0,w-3,x-1,y,z)-2*q(0,w-3,x+1,y,z)+
    q(1,w-2,x+2,y,z)+q(1,w-2,x,y+2,z)+q(1,w-2,x,y,z+2)+
    q(1,w-2,x-2,y,z)+q(1,w-2,x,y-2,z)+q(1,w-2,x,y,z-2)+
    (0-4)*q(1,w-2,x,y,z)-q(1,w-4,x,y,z)
 2->2*q(0,w-1,x,y+1,z)-2*q(0,w-1,x,y-1,z)+
    2*q(0,w-3,x,y-1,z)-2*q(0,w-3,x,y+1,z)+
    q(2,w-2,x+2,y,z)+q(2,w-2,x,y+2,z)+q(2,w-2,x,y,z+2)+
    q(2,w-2,x-2,y,z)+q(2,w-2,x,y-2,z)+q(2,w-2,x,y,z-2)+
    (0-4)*q(2,w-2,x,y,z)-q(2,w-4,x,y,z)
 3->2*q(0,w-1,x,y,z+1)-2*q(0,w-1,x,y,z-1)+
    2*q(0,w-3,x,y,z-1)-2*q(0,w-3,x,y,z+1)+
    q(3,w-2,x+2,y,z)+q(3,w-2,x,y+2,z)+q(3,w-2,x,y,z+2)+
    q(3,w-2,x-2,y,z)+q(3,w-2,x,y-2,z)+q(3,w-2,x,y,z-2)+
    (0-4)*q(3,w-2,x,y,z)-q(3,w-4,x,y,z)
 else if (v,w,x,y,z)==(0,0,0,0,0) then 1 else 0
```